**FOR FIN 4310 MANAGERIAL FINANCE**

**LECTURE 10**

***FINANCIAL ANALYSIS, CORPORATE VALUATION AND CAPITAL FORMATION***

by

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**LECTURE 10. INVESTING UNDER CONDITIONS OF UNCERTAINTY**

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**10.1. Introduction**

The myriad of outcomes of predicted cash flows in any given year makes it problematical to have a single cash flow prediction for an investment, whether in securities like stocks and their dividend streams, or non-securities like capital projects and their cash flow streams. At best the cash flow predicted in any given year is an expected value of the possible cash flows for that year, making values, NPV and IRR themselves expected values of the net present value (EVNPV) and expected values of the internal rate of return (EVIRR). The various outcomes of cash flows in any given year may be driven by different scenarios such as what the cash flow outcomes would be with a good economy, an average economy or a bad economy. These may also be affected by various sensitivities, such as what would happen to a NPV or IRR if revenues rose or fell by 10%, if costs rose or fell by 10%, if the cost of capital rose or fell by 1%, and so on. As such, Scenario and Sensitivity Analysis can lead to a distribution of possible NPVs and IRRs for investments from which their expected values may be extracted. Another avenue by which such a distribution may be seen is simulations, where all of the key variables leading to a cash flow or cost of capital determination are allowed to change based on a random selection of their values given the existence of mean and standard deviation parameters for each. Such a Monte Carlo Analysis may lead with use of computer programs to over 100,000 possible NPVs and IRRs, from which the expected values are extracted.

The cash flow in any given year, whether obtained from Scenario, Sensitivity or Monte Carlo Analysis, may not only have various possible outcomes for any given year, but may also be inter-temporally perfectly, imperfectly or not correlated. It is possible that good (bad) outcomes in one year may result in relatively good (bad) outcomes in subsequent years, that these be somewhat but not perfectly related, or that the yearly outcomes be unrelated at all. An analysis of investments under uncertainly needs to take these possibilities into consideration.

One final point to be mentioned is that when EVNPV’s are explicitly calculated based on an underlying distribution of NPVs from an associated distribution of cash flows, the relevant discount rate to be used is the risk-free rate if the standard deviation of the EVNPV is used in the analysis of risk. The risk-free rate is used to obtain the NPVs, because risk is explicitly considered through the standard deviations of the EVNPV, such that using a risk-adjusted discount rate would double count risk. The evaluations of the project in this case is based on probabilities that projects will have negative NPV’s or have IRRs below that of the risk free rate. If standard deviations of the EVNPV or EVIRR are not considered in the analysis, then it is appropriate to use the weighted average cost of capital as the discount rate. These points will be illustrated later in this chapter.

# **10.2. Imperfectly Correlated and Uncertain Cash Flows**

Consider a project in Ireland that has the pattern of imperfectly correlated Euro cash flows from time periods 0 to 2 shown in the decision tree in Table 10.1 below. The cash outflow of Euro 3,000 at time period 0 has 100% probability, as it is known with certainty. The possible cash flows at time period 1 are Euro 2,000 with 40% initial probability and Euro 3,000 with 60% initial probability. The possible cash flows at time period 2 are conditional on the cash flow outcome in time period 1. If the cash flow in time period 1 is Euro 2,000, then possible cash flows at time period 2 are Euro 1,000 with 40% conditional probability and Euro 2,000 with 60% conditional probability. If the cash flow in time period 1 is Euro 3,000, then possible cash flows at time period 2 are Euro 3,000 with 40% conditional probability and Euro 4,000 with 60% conditional probability.

**Table 10.1. Decision Tree for Imperfectly Correlated Intertemporal Cash Flows**

Periods

Zero One Two

Initial Conditional

CF0 CF1 Probabilities CF2 Probabilities

EURO 1,000 .40

EURO 2,000 .40

EURO 2,000 .60

-EURO 3,000

EURO 3,000 .40

EURO 3,000 .60

EURO 4,000 .60

**10.3. Expected Value and Standard Deviation of Net Present Values for Decision Tree for Imperfectly Correlated Intertemporal Cash Flows with Risk-free Rate**

The EVNPV is calculated next. It is assumed that the risk-free discount rate in Ireland is 5%. First calculate the NPVi for each possible pattern of cash flow i using the 5% discount rate.

The first cash flow pattern in Table 10.1 is –EURO 3,000 at time 0, +EURO 2,000 at time 1 and +EURO 1,000 at time 2. The NPV pertaining to this cash flow pattern is –EURO 188.21, as shown in the first row under the column NPVI below in Table 10.2.

The second cash flow pattern is –EURO 3,000 at time 0, +EURO 2,000 at time 1 and +EURO 2,000 at time 2. The NPV pertaining to this cash flow pattern is EURO 718.82, as shown in Table 10.2..

The third cash flow pattern is –EURO 3,000 at time 0, +EURO 3,000 at time 1 and +EURO 3,000 at time 2, and the fourth pattern is –EURO 3,000 at time 0, +EURO 3,000 at time 1 and +EURO 4,000 at time 2. These cash flow patterns yield NPV’s of EURO 2,578.23 and EURO 3,485.26, respectively.

The joint probabilities for each of these four cash flow patterns are the product of the initial probabilities and conditional probabilities for each respective pattern. The conditional probabilities are the probability of the second cash flow occurring, given that the first cash flow occurs. The joint probabilities are shown in the next to last column in Table 10.2.

**Table 10.2. Expected Value of Net Present Values for Decision Tree for Imperfectly Correlated Intertemporal Cash Flows, using 5% Risk Free Rate**

Initial Conditional Joint

CF0 CF1 Probabilities CF2 Probabilities Probabilities NPVi

EURO 1,000 .40 .16 -EURO 188.21

EURO 2,000 .40

EURO 2,000 .60 .24 EURO 718.82

-EURO 3,000

EURO 3,000 .40 .24 EURO 2,578.23

EURO 3,000 .60

EURO 4,000 .60 .36 EURO 3,485.26

The EVNPV is equal to the weighted average of the NPVi, where each is weighed by its joint probability. In this example, the EVNPV is EURO 2015.87, equal to EVNPV = EURO 2015.87

= .16 (-EURO 188.21) +.24 (EURO 718.82)

+ .24 (EURO 2,578.23) + .36 (EURO 3,485.26)

= -EURO 30.11 + EURO 172.52 + EURO 618.77 + EURO 1,254.69

The risk of this project is measured by the standard deviation of its EVNPV (σ EVNPV) using the standard formula, such that

σ EVNPV = EURO 1,426.25

= [.16 (-EURO 188.21 - EURO 2015.87)2

+ .24 (EURO 718.82 - EURO 2015.87)2

+ .24 (EURO 2,578.23 - EURO 2015.87) 2

+ .36 (EURO 3,485.26 - EURO 2015.87) 2] ½

= [EURO 2,034,214.48] ½

It is useful and convenient to analyze in combination this project’s expected value and risk. One way is to calculate its coefficient of variation (σ EVNPV / EVNPV), which in this case equals 0.7075 (or EURO 1,426.25 / EURO 2015.87). When more than one project is analyzed, they could be ranked accordingly, with the lower coefficient of variation one better.

Even more useful is to utilize these statistics to calculate the probability that the actual net present value of the project would be below zero, where it is desirable to have projects with low probabilities of yielding negative results. In this case, calculate the Z-statistic as follows,

Z = (0 – EVNPV) / STDEVNPV

= (0 - EURO 2015.87) / EURO 1,426.25

= -1.41

and from a normal probability distribution table, obtain a 10.20 percent probability that the actual net present value of the project would be below zero (equivalent to the probability that Z is less than 1.41), such that

Probability Z < -1.41 is 10.20%

When more than one project is analyzed, they could be ranked accordingly, with the one with the lowest probability that Z is negative being higher ranked. These calculations rely on a normal distribution assumption for the NPVi ‘s of the investment, although the calculation of this probability is possible without this assumption, using instead Chebychev’s inequality theorem.

**10.4. Expected Value and Standard Deviation of Internal Rates of Return for Decision Tree for Imperfectly Correlated Intertemporal Cash Flows**

The EVIRR can be calculated in a similar fashion as the EVNPV. First calculate the IRRi for each possible pattern of cash flow i. Using the cash flow patterns previously described and shown in Table 10.1, the IRRi‘s for each possible pattern of cash flow i’s are 0%, 21.52%, 61.80% and 75.83% as shown in the column IRRi below in Table 10.3.

**Table 10.3. Expected Value of Net Present Values for Decision Tree for Imperfectly Correlated Intertemporal Cash Flows, using 5% Risk Free Rate**

Initial Conditional Joint

CF0 CF1 Probabilities CF2 Probabilities Probabilities IRRi

EURO 1,000 .40 .16 0%

EURO 2,000 .40

EURO 2,000 .60 .24 21.52%

-EURO 3,000

EURO 3,000 .40 .24 61.80%

EURO 3,000 .60

EURO 4,000 .60 .36 75.83%

The EVIRR is equal to the weighted average of the IRRi, where each is weighed by its joint probability. The EVIRR is 47.30%, equal to

EVIRR = 47.30%

= .16 (0%) +.24 (21.52%) + .24 (61.80%) + .36 (75.83%)

The risk of this project can now be measured by the standard deviation of its EVIRR (σEVIRR) using the standard formula, such that

σ EVNPV = EURO 1,426.25

= [.16 (0.0000 – 0.4730)2

+ .24 (0.2152 – 0.4730)2

+ .24 (0.6180 – 0.4730) 2

+ .36 (0.7583 – 0.4730) 2] ½

= [(0.086095834)] ½

= 0.293420916 = 29.34%

Using these mean and standard deviation values for the EVIRR, calculate the Z-statistic (using the risk free rate as the critical value) as follows,

Z = (Risk free rate – EVIRR) / STDEVIRR

= (0.05 – 0.4730) / 0.2934

= -1.44

and from a normal probability distribution table, obtain a 9.6% probability that the actual IRR of the project would be below the 5% risk free rate (equivalent to the probability that Z is less than 1.44), such that

Probability Z < -1.44 is 9.6%

Alternatively, the risk of the project could be measured by adding a premium to the risk free rate, to obtain its risk adjusted discount rate, which in our example is assumed to equal 9%. In this case, as the EVIRR of 47.3% is greater than its risk adjusted 9% rate, the investment is acceptable.

**10.5. Expected Value of Net Present Values for Decision Tree for Imperfectly Correlated Intertemporal Cash Flows with Risk-adjusted Discount Rate**

Assume that the risk-adjusted discount rate in Ireland is 9%. Now, only the NPVi for each possible pattern of cash flow, and the EVNPV for the investment, is calculated when using a risk-adjusted discount rate. No standard deviations, Z-values or probabilities that the investment’s NPV is below zero are calculated, as risk is accounted for through the discount rate. Using the 9% discount rate, the NPVi are shown below in Table 10.4.

**Table 10.4. Expected Value of Net Present Values for Decision Tree for Imperfectly Correlated Intertemporal Cash Flows, using 9% Risk Adjusted Discount Rate**

Initial Conditional Joint

CF0 CF1 Probabilities CF2 Probabilities Probabilities NPVi

EURO 1,000 .40 .16 -EURO 323.45

EURO 2,000 .40

EURO 2,000 .60 .24 EURO 518.22

-EURO 3,000

EURO 3,000 .40 .24 EURO 2,277.33

EURO 3,000 .60

EURO 4,000 .60 .36 EURO 3,119.01

Notice as expected that the NPVi ‘s with a 9% risk adjusted discount rate are less than with a 5% risk free rate. The EVNPV is equal to the weighted average of the NPVi, where each is weighed by its joint probability. In this example, the EVNPV is EURO 1,742.02, equal to

EVNPV = EURO 1,742.02

= .16 (-EURO 323.45) +.24 (EURO 518.22)

+ .24 (EURO 2,277.33) + .36 (EURO 3,119.01)

The EVNPV of EURO 1,742.02 is less than the EVNPV of EURO 2015.87 with the risk free rate, such that the risk adjusted discount rate penalizes the project for its risk. As such, there is no need to consider risk through the standard deviation of the projects’ NPVi‘s or the probability that the NPV would fall below zero. In this case, one would accept the project as its EVNPV is greater than zero.

**10.6 Expected Value of Net Present Value and Standard Deviation for Portfolios of Projects**

What happens when a firm combines projects, therefore creating a portfolio of projects? One may observe driving through Colorado resorts at high-altitude that not only have downhill ski facilities but also golf courses. It is obvious that in the winter the golf course and in the summer the ski facilities would be underutilized. Stated another way, in the winter the ski facilities would produce relatively high cash flow and the gold course relatively low cash flow, and that the opposite would occur in the summer. The resorts creating this type of portfolio have assets that are negatively correlated in terms of performance, and portfolios that produce a steadier, less risky cash flow. Resorts that only have ski facilities or golf courses would experience greater variance in cash flow than those with a combination of these. The essence of portfolio diversification is to have multiple projects or investments with each having low or negative correlation with the others to benefit the most from risk reduction while maintaining return. This section illustrates these concepts using standard formulas in portfolio analysis regarding their return and risk.

Assume that the financial manager for a ski resort known as “Downhell” is assigned the task of analyzing the financial feasibility of adding indoor tennis courts to the resort’s already existing skiing and ice-skating facilities. The expected values of net present values (EVNPV) and standard deviations (σEVNPV) of the existing facilities and the proposed tennis facility have already been calculated and are given below.

EVNPV ($) σEVNPV ($)

Skiing (existing) 100 20

Ice-skating (existing) 50 15

Tennis (proposed) 10 3

As an aside, notice that in the case of projects, the amounts shown for the EVNPV are in dollar terms, unlike for securities where returns are shown in percentage terms. The dollar values for the projects therefore directly reflect their weights in the portfolio, making it unnecessary to use percentage weights.

The financial manager must approach this assignment utilizing a portfolio analysis paradigm, as the resort considers adding tennis to the existing skiing and ice-skating portfolio. Assume that the correlation coefficients between the existing and proposed facilities have already been calculated as shown below.

1. Skiing 2. Ice-skating 3. Indoor Tennis

1. Skiing +1.0

2. Ice-skating +0.80 +1.0

3. Indoor Tennis -0.50 -0.25 +1.0

Initially of interest are the value and risk of the existing portfolio of projects for “Downhell”. Subsequently, we will compute and compare the EVNPV and the σEVNPV for the combination of existing facilities (Skiing and Ice-skating) to the EVNPV and the σ for the combination of existing (Skiing and Ice-skating) plus the proposed (Indoor Tennis) facilities to decide whether the latter is beneficial in terms of return and risk.

In the solutions shown below, we use the notation (1) to represent Skiing, (2) to represent Ice-skating, and (3) to represent Indoor Tennis. The EVNPV and the σEVNPV for the combination of existing facilities [Skiing and Ice-skating, or (1) and (2)] are:

EVNPV1+2 = EVNPV1 + EVNPV2 = 100 + 50 = $150

σ1+2 = [σ21 + σ22 + 2 σ1 σ2 ρ1,2 ] 1/2  = [202 + 152 + 2 (20) (15) (0.80)] 1/2

= [400 + 225 + 480] ½ = [1105] ½ = $33.24

where σ21 and σ22 are the variances of the expected value of net present value for the Skiing and Ice-skating facilities, respectively, and σ1 σ2 ρ1,2 is their covariance, where ρ1,2 is the correlation coefficient between the values of the Skiing and Ice-skating facilities.

Notice that the Skiing and Ice-skating facilities have a high 0.80 correlation, which means that the portfolio of these two projects do not produce much in terms of risk reduction. The value of the two projects combined is simply the summation of each of their expected values of net present values. The risk for the portfolio (standard deviation of the portfolio) considers each facility’s own standard deviation and their covariance (considering their correlation). Notice that the portfolio’s risk, that is σ1+2, is 33.24, which is not much lower than the summation of their standard deviations which equals 35, that is 20 plus 15, owing to their high correlation. There are no weights to consider as the dollar values of the returns and risk explicitly account for the weights.

The EVNPV and σEVNPV for the combination of existing facilities (Skiing and Ice-skating) plus the proposed Indoor Tennis facility is:

EVNPV1+2+3 = EVNPV1 + EVNPV2 + EVNPV3 = 100 + 50 + 10 = $160

σ1+2+3 = [σ21 + σ22 + σ23 + 2 σ1 σ2 ρ1,2 + 2 σ1 σ3 ρ1,3 + 2 σ2 σ3 ρ2,3 ] 1/2

= [202 + 152 + 32 + 2 (20) (15) (0.80) + 2 (20) (3) (-0.50) + 2 (15) (3) (-0.25)] 1/2

= [400 + 225 + 9 + 480 + (-60) + (-22.5)] ½

= [1031.5] ½

= $32.11

An alternative approach for calculating σ1+2+3 that takes into account the calculations already made for σ1+2 of 33.24 for the standard deviation of the Skiing and Ice-skating facilities is shown below, such that:

σ1+2+3 = = [σ21+2 + σ23 + 2 σ1 σ3 ρ1,3 + 2 σ2 σ3 ρ2,3 ] 1/2

= [33.242 + 32 + 2 (20) (3) (-0.50) + 2 (15) (3) (-0.25)] 1/2

= [1104.90 + 9 + (-60) + (-22.5)] ½

= [1031.5] ½

= $32.11

Adding the Indoor Tennis facility to the existing facilities increases the EVNP from $150 to $160 while reducing the standard deviation from $33.24 to $32.11. Thus, one should definitely accept the Indoor Tennis Court facility because it adds value ($160 versus $150) to the portfolio while reducing its risk ($32.11 versus $33.24). The primary reason for the risk reduction is that the Indoor Tennis Court facility has low correlations with “Downhell’s” existing facilities – a correlation of -0.50 with Skiing and

-0.25 with Ice-skating.

**10.6.1. Example 1**

A bookstore has three books in its portfolio of books, titled The Pleasures of Finance, Transcendental Meditation, and Beyond the Stars. The expected values of net present values (EVNPV) and standard deviations (σEVNPV) for each of these three books are shown below.

EVNPV ($) σ ($)

The Pleasures of Finance (PF) 1,000 100

Transcendental Meditation (TM) 800 200

Beyond the Stars (BS) 1,500 600

Also, the correlation coefficients among these three books are shown below.

1. PF 2. TM 3. BS

1. PF +1.0

2. TM -0.20 +1.0

3. BS -0.40 +0.50 +1.0

Question 1: Determine the EVNPV and the σEVNPV only for The Pleasures of Finance (PF) and Transcendental Meditation (TM) book combination.

Answer 1: Note that in the answers, the notation (1) represents The Pleasures of Finance, (2) represents Transcendental Meditation, and (3) represents Beyond the Stars. The EVNPV and the σEVNPV only for The Pleasures of Finance (PF) and Transcendental Meditation (TM) book combination are:

EVNPV1+2 = EVNPV1 + EVNPV2 = 1,000 + 800 = $1,800

σ1+2 = [σ21 + σ22 + 2 σ1 σ2 ρ1,2 ] 1/2

= [1002 + 2002 + 2 (100) (200) (-0.20)] ½

= [42,000] ½ = $204.94

In this case, the t (or Z) value is -8.78 = (0 – 1,800) / 204.94

Question 2: Examine how the EVNPV and the σEVNPV changes when the book Beyond the Stars is added to the previous combination of The Pleasures of Finance (PF) and Transcendental Meditation (TM).

Answer 2: The EVNPV and σ for the combination of all three books is:

EVNPV1+2+3 = EVNPV1 + EVNPV2 + EVNPV3 = 1,000 + 800 + 1,500 = $3,300

σ1+2+3 = [σ21 + σ22 + σ23 + 2 σ1 σ2 ρ1,2 + 2 σ1 σ3 ρ1,3 + 2 σ2 σ3 ρ2,3 ] 1/2

= [1002 + 2002 + 6002 + 2 (100) (200) (-0.20) + 2 (100) (600) (-0.40) + 2 (200) (600) (0.50)] 1/2

= [474,000] ½ = $688.48

or alternatively

σ1+2+3 = = [σ21+2 + σ23 + 2 σ1 σ3 ρ1,3 + 2 σ2 σ3 ρ2,3 ] 1/2

= [42,0002 + 6002 + 2 (100) (600) (-0.40) + 2 (200) (600) (0.50)] 1/2

= [474,000] ½ = $688.48

In this case, the t (or Z) value is -4.79 = (0 – 3,300) / 688.48

Based on the results, the combination of The Pleasures of Finance and Transcendental Meditation is better than the combination of all three books, for the former has a lower probability (higher t or Z value in absolute value) that a negative NPV will occur. That is to say, even though Beyond the Stars contributes $1,500 in value to the existing portfolio, its high stand-alone risk (its standard deviation is $600) and high correlation with the other books suggests that its contribution to risk outstrips its contribution to return. As a result, the portfolio of all three books has a higher probability that a negative NPV will occur than the portfolio of only the two books being considered.